Journal of Economic Behavior & Organization
Vol. 47 (2002) 309–333

JOURNAL OF
Economic Behavior
& Organization

www.elsevier.com/locate/econbase

Locally interdependent preferences in a general equilibrium environment

Ann Maria Bell

NASA Ames Research Center, Mail Stop 239-8, Moffett Field, CA 94305-1000, USA

Received 30 March 1999; received in revised form 12 October 1999; accepted 19 October 1999

Abstract

This paper explores the consequences of interdependent preferences for consumer goods, that is, preferences that evolve in response to the consumption decisions of neighboring agents. The key feature is that the interdependence of preferences coexists and interacts with the price mechanism in a general equilibrium environment. The interaction between the negative feedback operating through the price system and the positive feedback expressed in the bandwagon effect creates distinct geographic patterns of consumption on the micro-level and a characteristic evolution of average preferences and production on the macro-level. In equilibrium, agents' preferences and consumption are completely polarized into stable regions in which every agent consumes the same good exclusively. © 2002 Elsevier Science B.V. All rights reserved.

JEL classification: D83; R13; D51

Keywords: Interdependent preferences; Bandwagon effect; Herding; Network externalities

1. Introduction

As any parent or marketing specialist can confirm, an individual's preferences for a good are often influenced by other people's consumption choices. No fashion is so outrageous that repeated exposure to it cannot persuade some segment of the population of its desirability. Preferences for goods such as pungent cheeses or opera are often referred to as "acquired tastes". The prevalence of "product placement", a promotion tactic which involves use of branded consumer items as background props in movies and television programs, suggests that exposure to other people's consumption effectively sways consumer opinion.

This paper explores the individual and aggregate consequences of a bandwagon effect, or a process of "acquiring tastes" in which preferences for a good increase because other

E-mail address: abell@mail.arc.nasa.gov (A.M. Bell).

PII: S0167-2681(01)00177-9

agents have purchased it. The behavioral assumption is straightforward: the more agents see of a good, the more they like it. Agents do not attempt to “keep up with the Jones,” nor do they wish to conform to their neighbors’ consumption patterns, [1] rather, their tastes passively respond to repeated exposure to a good. The interdependence of preferences and observed consumption is local and spatially-based, occurring between agents in fixed reference groups. For example, the reference groups might be teenagers involved in the same high school activities or doctors practicing in the same hospital or clinic.

A bandwagon effect, or imitation in preferences, can also result from increasing returns to consumption across agents or from network externalities in consumption. For example, neighbors who play a similar style of music can critique each others’ performance, and users of the same software packages can exchange files easily and share information about short-cuts or bugs. Consequently, agents may alter their preferences to reflect their neighbors’ previous consumption decisions. A historical example is the “QWERTY” layout of key-boards discussed by David (1986). Arthur (1989) utilizes VHS and Beta video formats as an example of goods with network externalities or increasing returns to adoption across agents. Leibenstein (1970) provides an early treatment of a bandwagon effect that results from network externalities in consumption.

This paper shows how a bandwagon effect, or imitation in preferences, interacts with traditional market-based forces to create distinct geographic patterns of consumption on the micro-level and a characteristic evolution of average preferences and production on the macro-level. In equilibrium, agents’ preferences and consumption are completely polarized into stable regions in which every agent consumes the same good exclusively. The production technology is the key determinant of the evolution of aggregate quantities such as the price; the initial configuration of preferences typically plays a subordinate role. In particular, when production exhibits constant or decreasing returns to scale, average preferences evolve so as to minimize the per unit cost of production or, equivalently, to maximize the total quantity of goods produced from a fixed input. In an exchange economy, the evolution of average preferences reflects the availability of goods: greater relative availability of a good causes more agents to prefer that good exclusively in equilibrium. The initial configuration of preferences is the key determinant of the evolution of preferences only when the two goods are equally easy to produce with a constant returns to scale technology.

1.1. Global interactions and general equilibrium

One of the critical features that distinguishes the models presented here from previous treatments of interacting preferences is that the interaction of preferences and observed consumption occurs in the context of a competitive general equilibrium environment. Agents interact globally in a centralized market for goods and locally through the interdependence of agents’ preferences and neighborhood consumption.

[1] Several recent papers model the desire for conformity. Bernheim (1994) considers how uniform behavior can arise despite heterogeneous preferences for actions when agents care about other’s perceptions of their preferences. (Kuran, 1987, 1989) demonstrates that large, sudden changes in group behavior can occur when people conceal their preferences in order to (apparently) conform with the prevailing regime. (Jones, 1984) also examines the economic consequences of conformism.

As in any market economy, the price acts as a negative feedback mechanism that limits consumption of scarce or highly desirable goods and encourages consumption of plentiful or less desirable goods. The strength of this negative feedback depends, of course, on the elasticity of supply with respect to price. In contrast, the changes in tastes based on exposure create a positive feedback mechanism that increases agents' preferences for a good whenever agents in their local neighborhood consume more of it relative to other goods. The interaction between the two forces creates a sharp contrast between the behavior of aggregate and individual preferences.

Because future preferences depend positively on total neighborhood consumption of the goods, agents who prefer a less expensive or relatively abundant good have a greater influence on the local adaptation of preferences. In essence, agents "learn to love" cheap goods because they are abundant and more likely to be observed. Consequently, the economy as a whole tends towards a global mean of preferences which minimizes the cost of production or which reflects the available supply of goods. The abundance of potatoes in Ireland or of fast food restaurants in the United States, for example, would cause more individuals to prefer those foods over time as a result of repeated observation of other agents' consumption. Exposure, or word of mouth advertising, reorients consumers' preferences towards the cheaper good. Equivalently, agents lose interest in or disprefer goods that they do not observe in their neighborhoods, hence, average preferences tend away from goods that are scarce and/or expensive. This occurs even if agents' initial preferences strongly favor the scarcer good. Consequently, the number of agents consuming a good in the steady state is proportional to the availability of the good or the cost of producing it. However, when both goods are equally easy to produce and changes in demand have no effect on the price, the centralized market interactions do not influence the evolution of average preferences, since any global mean of preferences minimizes the total cost of production.

A number of recent papers examine fads, epidemics or "herd effects" in consumption in terms of the transmission of information (e.g. Arthur and Lane (1992), Banerjee (1992), Bikchandani et al. (1992), and Kirman (1993)). However, none of these informationally based models account for the effect of changing prices in generating and sustaining fads in consumption nor do they consider the spatial dimensions of consumption behavior. Both in spirit and design, the model presented here is most similar to Pollak's (1976) analysis of interdependent preferences. Pollak's rule for updating preferences depends directly on other agents' preferences, consequently, prices and the availability of goods cannot influence the evolution of preferences. Bala and Long (1998) start from the assumption that preferences evolve in favor of cheaper goods and analyze the resulting trajectory. In contrast, here the evolution of average preferences towards the good which is relatively abundant or cheaper to produce demonstrated is a direct consequence of the positive interdependence of preferences across agents, not an assumption of the model.

1.2. Local interactions and persistent spatial heterogeneity

Another key result is that changes in tastes that arise from local exposure to a good lead to stable regional patterns of consumption. Steady state preferences and consumption are polarized: clusters of consumers who prefer one good exclusively exist along side clusters of consumers who prefer the other good exclusively. Given the assumption that preferences adapt locally, this local polarization is perhaps not surprising. However, the relative frequency and type of clusters, and hence average preferences and total production, depend crucially on the available production technology and on the general equilibrium market structure. Furthermore, the stability of regional variations in consumption and the relative unimportance of the initial state of agent's preferences in these models contrasts sharply with the results from nonmarket-based models discussed above. [2] A large number of economic and sociological behaviors exhibit distinct spatial variation. The results presented here suggest that local interdependence of preferences combined with global market interactions may provide an explanation for the existence and long term stability of regional variations in tastes and consumption.

Among the sociological and economic phenomena that exhibit persistent spatial heterogeneity, regional variations in medical practice are both well-documented and difficult to explain in terms of traditional models of optimizing behavior. A brief search of the MEDLINE database for the years 1989 to 1994 located over 20 articles documenting geographic variations in physicians' practice. Geographic variation in the use of breast-conserving surgery, for example, persists even after controlling for hospital and patient characteristics (Nattinger et al., 1992; Farrow et al., 1992). In a review of international and regional variations in the use of health care services, Blais (1989) notes that the differences cannot be adequately explained by "errors of data, population characteristics, or health care system features" and instead attributes them to "professional uncertainty" about the efficacy of alternative treatments. [3] The results of this paper suggest that stable, long term regional variations in medical practice may persist despite globally available information that indicates the superiority of one treatment over another. Similar regional patterns emerge in legal practices such as the length of time for scheduling of hearings and trials, the use of plea bargains and the severity of punishments for similar crimes (Church, 1985; Braucher, 1993; Kritzer and Zemans, 1993; Sullivan et al., 1994). Church (1985) notes that lawyers and judges practicing in the same court tend to share the same "established expectations, practices and informal rules of behavior" identified with a "local legal culture". The local interdependence of expectations and norms consequently leads to persistent regional variations in legal practice.

Glaeser et al. (1996) analyze the spatial and temporal variation in crime rates using an empirical model of social interactions. Topa (1997) investigates the role of local spillovers and social interactions in generating positive spatial correlations in unemployment and tests the predictions of the model using Census tract data. Similarly, differences in tobacco use between men and women, and across geographic areas exhibit persistent regional variations in consumption (Shopland et al., 1992; Feder, 1996).

The models and corresponding analysis utilize techniques from nonlinear dynamical systems theory. Related nonlinear models of local interactions and the spatial distribution of economic activity are presented in Föllmer (1974), which analyzes the general equilibrium properties of an economy with random, interacting preferences; in Durlauf (1995), which examines the relationship between increasing returns to education, neighborhood formation

[2] Arthur and Lane (1992), Banerjee (1992), Bikchandani et al. (1992), and Kirman (1993).

[3] The same situation prevails if the physicians' concern is justifying their treatment choices in case of a malpractice suit.

and the intergenerational transmission of income inequality; and in Brock (1994) which explores the local transmission of information and asset prices.

Section 2 describes the models and motivates the specification of the bandwagon effect. Section 3 discusses a sample simulation of one of the models and presents examples of the equilibria of the models. Section 4 examines the equilibrium characteristics of neighborhoods and shows how they can be understood in terms of simple rules. Section 5 analyzes the stability of equilibria. Section 6 discusses the aggregate properties of the models. Section 7 considers the possibility of strategic behavior in agents' consumption decision. Section 8 concludes.

2. Model description

There are a large number of agents located on a lattice structure or arranged in a rectangular array. An agent's neighborhood consists of the agent and the eight adjacent agents on the lattice. In order to complete the neighborhoods of the agents on the edges of the lattice, the lattice is connected top to bottom and side to side, forming a torus, or periodic boundary condition.

2.1. General equilibrium

There are two consumption goods, x_1 and x_2, measured in comparable units (i.e. x_1 is red t-shirts, x_2 is blue t-shirts). Three alternative technologies are considered: an exchange economy where the supply of goods is fixed, constant returns to scale production and decreasing returns to scale production.

Each consumer has a unit income elastic Cobb–Douglas utility function and an endowment of goods or of labor and partial ownership of the firms. The parameter a, which determines the relative contribution of consumption of x_1 to a consumer's utility, varies both across agents and over time. Agents are indexed by (i, j), representing their location in the lattice. The superscript (I, J) is used when necessary to distinguish a particular individual agent, the superscript (i, j) is used to specify summation over more than one agent or over the entire lattice. The subscript $\text{nbhd}_{(I,J)}$ denotes the nine agent neighborhood that includes agent (I, J) and the eight nearest neighbors. There is no storage of goods and no credit or futures markets. Note that agents do not account for the influence of their consumption choices today on the future trajectory of their preferences. The consequences of this assumption are discussed in Section 7. Agents maximize utility period by period subject to a budget constraint that corresponds to the type of technology employed:

$$\begin{aligned} &\max u(x_1, x_2) = x_1^{a_t} x_2^{1-a_t}; \quad 0 \le a_t \le 1 \\ &\text{s.t. } p_1 x_1 + p_2 x_2 = p_1 e_1 + p_2 e_2 \ \text{ or } \ \text{s.t. } p_1 x_1 + p_2 x_2 = wL + \frac{\pi_1 + \pi_2}{N}, \end{aligned} \tag{1}$$

where e_1 and e_2 in the first budget constraint are the agent's endowments of good 1 and good 2 (assumed to be the same for all agents), and L in the second budget constraint is the agent's endowment of labor (also assumed to be the same for all agents), w is the wage or price of labor, π_1 and π_2 are total profits for the firms, and N is the number of consumers.

The first budget constraint is used for an endowment economy, the second for a production economy.[4] The (I, J) superscripts are suppressed when it is not necessary to distinguish agents. Initial preferences, or the $a^{I,J}$'s for all agents in the first period, are generated randomly from an identical and independent distribution. Agents' demand functions are:

$$x_1(p_1, p_2) = a_t\left(e_1 + \frac{p_2}{p_1}e_2\right) \text{ or } x_1(p_1, p_2, w) = \frac{a_t}{p_1}\left(wL + \frac{\pi_1 + \pi_2}{N}\right),$$
$$x_2(p_1, p_2) = (1 - a_t)\frac{p_1}{p_2}\left(e_1 + \frac{p_2}{p_1}e_2\right)$$
$$\text{or } x_2(p_1, p_2, w) = \frac{1 - a_t}{p_2}\left(wL + \frac{\pi_1 + \pi_2}{N}\right). \tag{2}$$

There are two representative profit maximizing firms. Firms solve the following maximization problems when the available production technology exhibits constant returns to scale:

$$\begin{array}{ll} \max \pi_1 = p_1x_1 - wL_1 & \max \pi_2 = p_2x_2 - wL_2 \\ \text{s.t. } x_1 = b_1L_1 & \text{s.t. } x_2 = b_2L_2. \end{array} \tag{3}$$

Note that this technology is equivalent to allowing agents to costlessly convert their endowment of labor into their desired consumption bundle of goods 1 and 2. Firms solve the following maximization problems when the available production technology exhibits decreasing returns to scale:

$$\begin{array}{ll} \max \pi_1 = p_1x_1 - wL_1 & \max \pi_2 = p_2x_2 - wL_2 \\ \text{s.t. } x_1 = \sqrt{L_1} & \text{s.t. } x_2 = \sqrt{L_2}. \end{array} \tag{4}$$

The firms' demand and profit functions with decreasing returns to scale are:

$$x_1(p_1, p_2, w) = \frac{p_1}{2w}, \qquad x_2(p_1, p_2, w) = \frac{p_2}{2w},$$
$$L_1(p_1, p_2, w) = \frac{p_1^2}{4w^2}, \qquad L_2(p_1, p_2, w) = \frac{p_2^2}{4w^2},$$
$$\pi_1(p_1, p_2, w) = \frac{p_1^2}{4w}, \qquad \pi_2(p_1, p_2, w) = \frac{p_2^2}{4w}. \tag{5}$$

Agents trade in a global market with the equilibrium price determined through the imposition of the standard market clearing conditions. With exchange the market clearing price is:

$$\frac{p_1^*}{p_2^*} = \frac{e_2\sum_{i,j}a_t^{i,j}}{e_1\sum_{i,j}(1 - a_t^{i,j})}. \tag{6}$$

With constant returns to scale production the market clearing prices are:

$$\frac{p_1^*}{p_2^*} = \frac{b_2}{b_1} \quad \text{and} \quad w^* = p_1b_1 = p_2b_2. \tag{7}$$

[4] Note that profits are divided equally among all agents in this case.

With decreasing returns to scale production the market clearing prices are:

$$\frac{p_1^*}{p_2^*} = \sqrt{\frac{\sum_{i,j} a_t^{i,j}}{\sum_{i,j}(1 - a_t^{i,j})}} \quad \text{and} \quad w^* = \frac{1}{\sqrt{2\sum_{i,j}(1 - a_t^{i,j})(1 + NL)}}. \tag{8}$$

2.2. Updating preferences

After the equilibrium price, level of production and individual consumption bundles are determined, agents "look around" the neighborhood consisting of themselves and their eight nearest neighbors. If the amount of good 1 consumed in an agent's neighborhood this period is greater than 50% of the total neighborhood consumption then the agent's preferences for good 1 increase. Conversely, if the amount of good 1 consumed in the agent's neighborhood this period is less than 50% of the total neighborhood consumption then the agent's preferences for good 1 decrease. Agents adjust their preferences in direct proportion to the deviation of neighborhood consumption of x_1 from 50%. No special significance is attached to the consumption threshold of 50%; the use of other threshold values simply rescales the rule without affecting the dynamics.

The psychology of the rule is simple: the more an agent sees of a good, the more he or she prefers it. Exposure to neighborhood consumption or word of mouth advertising is the only factor that directly influences preferences. Agents continue to increase their preferences for a good even if their consumption already exceeds the neighborhood average: they do not attempt to match the neighborhood consumption bundle, nor are they influenced by any kind of "snob effect" or tendency to prefer more expensive goods on status grounds. [5]

The rule for updating preferences is:

$$a_{t+1}^{I,J} = a_t^{I,J} + r\left(\frac{\sum_{\text{nbhd}_{(I,J)}} x_1^{i,j}(p_1^*, p_2^*, w^*)}{\sum_{\text{nbhd}_{(I,J)}} x_1^{i,j}(p_1^*, p_2^*, w^*) + \sum_{\text{nbhd}_{(I,J)}} x_2^{i,j}(p_1^*, p_2^*, w^*)} - 0.5\right) \tag{9}$$

combined with the constraint

$$0 \le a_{t+1}^{I,J} \le 1. \tag{10}$$

The parameter r is a fixed stepsize, typically less than 1. By substituting equilibrium prices into demand functions and demand functions into Eq. (9) the transition rule can be expressed as a function of the neighborhood average of preferences and the price:

$$a_{t+1}^{I,J} = a_t^{I,J} + r\left(\frac{\sum_{\text{nbhd}_{(I,J)}} a_t^{i,j}}{\sum_{\text{nbhd}_{(I,J)}} a_t^{i,j} + (p_1/p_2)\sum_{\text{nbhd}_{(I,J)}}(1 - a_t^{i,j})} - 0.5\right) \tag{11}$$

[5] Not surprisingly, when agents attempt to match their future consumption with this period's neighborhood average by decreasing their preferences for a good when their consumption exceeds the neighborhood average, and vice versa, the system quickly converges to a steady state in which all agents have identical preferences and identical consumption bundles.

keeping in mind that the price typically depends on endowments, technological parameters, and the average value of preferences.

This specification depends on agents' purchases of goods, readily observable quantities, rather than on the unobservable preferences of other agents. A rule based on expenditures, another observable quantity, is a special case of Eq. (9): the update term of the rule reduces to $r((1/9)\sum_{\mathrm{nbhd}_{(I,J)}} a_t^{i,j} - 0.5)$ because $a^{i,j}$ is the fraction of wealth or expenditures devoted to good 1 with a Cobb–Douglas utility function, which is the same as the update term given in Eq. (11) with CRS production and $(p_1/p_2) = 1$. This also corresponds to direct observation of other agents' preferences.

The two goods must be measured in comparable units in order for Eq. (9) to be well defined. The economic motivation for a bandwagon effect or herding behavior requires slightly more: the goods in the model must fulfill the same function and compete directly with each other. For example, the two goods might be different brands of sneakers destined for teenage feet, competing academic journals in the same field, essentially identical formulations of shampoo packaged in differently shaped bottles, music lessons in different styles, e.g. folk versus classical guitar, different types of collectible porcelain figurines or competing word processing or spreadsheet packages. In the medical practice example, the two goods would be different procedures or treatments prescribed for the same condition; in the legal practice example, different plea bargains for the same crime. Without this comparability there is little reason to suppose that agents would be affected by observations of other's behavior.

After agents update their preferences, a new time period starts: agents begin with their new preferences and a fresh supply of endowment goods, choose their consumption bundles in the global marketplace, examine the consumption choices of the agents in their neighborhood, update their preferences accordingly, and so on.

Except for the random initial state of preferences these models are completely deterministic. The models are nonlinear dynamical systems with discrete time and space components but continuous state variables, i.e. coupled map lattices (Holden et al., 1992; Kaneko, 1992).

3. Simulation of the exchange economy

To facilitate an understanding of the dynamic behavior of the model and to motivate the analytical results concerning the existence and stability of equilibria presented below, a sample simulation of the exchange economy is presented in this section. This simulation illustrates the generic behavior of the system, or the behavior that results from randomly chosen initial conditions. More extensive simulation data are utilized in the discussion of the evolution of aggregate preferences, prices and total consumption.

Fig. 1 shows the evolution of the lattice of agents' preferences in the model of an exchange economy with $e_1 = e_2 = 1$, $r = 0.5$ and $N = 2500$. An agent with $a^{I,J} = 1$ is represented by a black square, an agent with $a^{I,J} = 0$ by a white square. Agents with $0 < a^{I,J} < 1$ are represented by the corresponding value on a gray scale. Note that the configuration of preferences, or of the $a^{I,J}$ parameters, completely determines the state of the system when all agents have identical endowments. The price and agents' consumption bundles can be calculated directly from preferences.

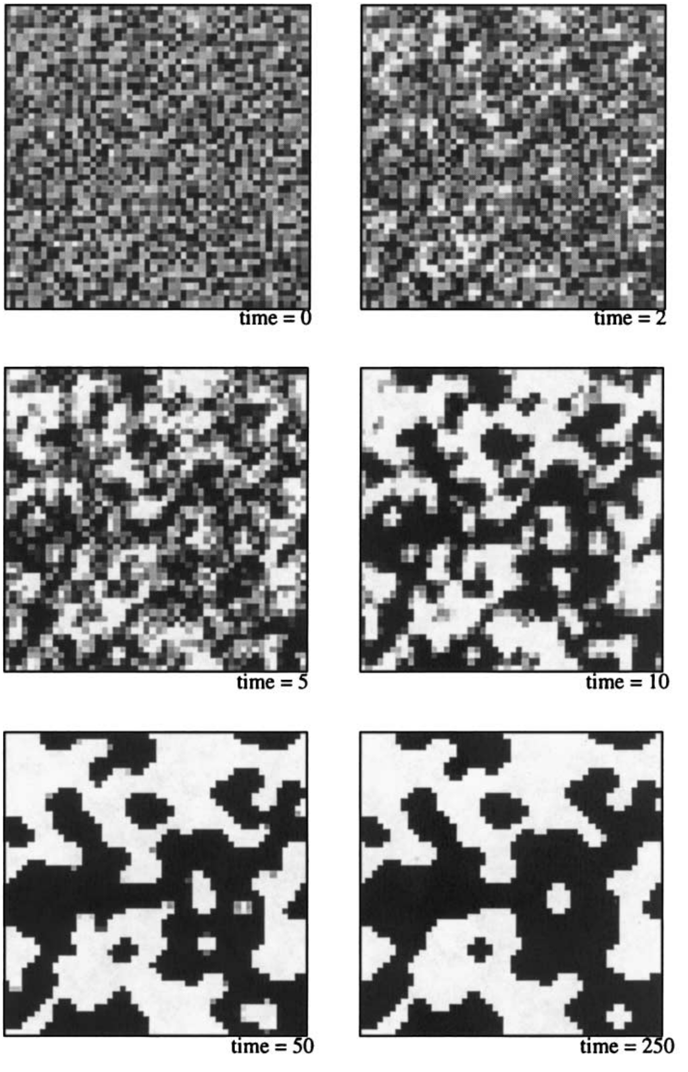

Fig. 1. Sample run of the exchange economy.

Starting from a randomly chosen initial state, clustering of preferences and consumption emerges rapidly and can be observed after only two iterations. After five iterations many of the agents have either $a^{I,J} = 0$ or $a^{I,J} = 1$ and consume one good exclusively. At time 10 the clusters of like-minded consumers are readily identifiable, and at time 50 almost all of the agents consume only one good. The lattice continues to evolve slowly as the edges of clusters are "smoothed" and isolated smaller groups of consumers are absorbed. Finally, an equilibrium is reached in which all agents have preferences equal to zero or one and consume one good exclusively.

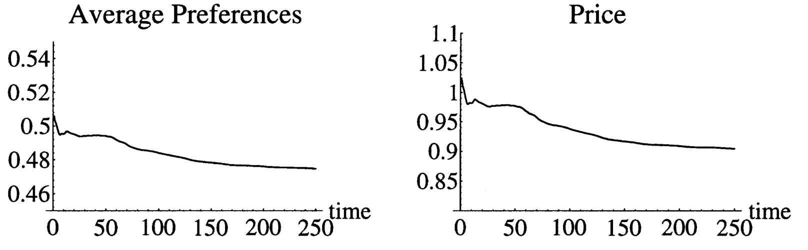

Fig. 2. Average preferences and price.

This process, the rapid formation of clusters, followed by a slow process of adjustments on the edges of the clusters, leading to an equilibrium composed exclusively of zeros and ones, typifies the behavior of the system for a wide variety of initial conditions and production technologies. In every simulation performed the lattice converged to an equilibrium where every agent's preferences were equal to either one or zero and every agent consumed one good exclusively. No limit cycles, random behaviors or multiperiod orbits were observed.

The evolution of average preferences over the lattice and the associated price is shown in Fig. 2. The initial mean of preferences is 0.506 and the final is 0.475. Although individual agents' preferences change dramatically, in this case average preferences stay close to the initial mean.

4. Conditions for and characteristics of steady states

4.1. Conditions for a steady state

Define the update term, or potential change in agent I, J's preferences, from the rule for updating preferences given by Eq. (11) as:

$$c_t^{I,J} = r\left(\frac{\sum_{\text{nbhd}_{(I,J)}} a_t^{i,j}}{\sum_{\text{nbhd}_{(I,J)}} a_t^{i,j} + (p_1/p_2)\sum_{\text{nbhd}_{(I,J)}}(1-a_t^{i,j})} - 0.5\right). \tag{12}$$

Recall that the price may depend on the relative economy-wide endowments of the goods, the available production technology, and average preferences. The lattice has achieved a steady state or is in equilibrium at time t if one of the following conditions holds for each (I, J):

$$\begin{array}{lll} \text{(C1)} & a_t^{I,J} = 1 & \text{and } c_t^{I,J} > 0 \\ \text{(C2)} & a_t^{I,J} = 0 & \text{and } c_t^{I,J} < 0 \\ \text{(C3)} & a_t^{I,J} = 1 & \text{and } c_t^{I,J} = 0 \\ \text{(C4)} & a_t^{I,J} = 0 & \text{and } c_t^{I,J} = 0 \\ \text{(C5)} & 0 < a_t^{I,J} < 1 & \text{and } c_t^{I,J} = 0. \end{array} \tag{13}$$

The conditions required for a steady state or equilibrium are the same with exchange and all types of production. Also, note that these conditions for a steady state are not limited to the rectangular lattice structure and nine person neighborhood: they characterize a steady state for all lattice structures and neighborhood definitions. In conditions C1 and C2 the update term $c_t^{I,J}$ is not identically equal to zero, but in conditions C3, C4 and C5 it is: this distinction plays an important role in the analysis of the stability of equilibria.

An equilibrium or steady state in which every agent's preferences equal zero or one exactly has polarized preferences and consumption: each agent consumes only one good in equilibrium. The equilibria in which every agents' preference parameter equals zero or in which every agents' preferences are equal to one are called trivial steady states. When an equilibrium exhibits polarized preferences the equilibrium conditions can be stated as a counting rule that depends on the number of agents in the neighborhood with $a^{I,J} = 1$ (the number of agents who consume good 1 exclusively) and the price. For example, when $(p_1/p_2) = 1$ an equilibrium exists when every agent with $a^{I,J} = 1$ is in a neighborhood consisting of five or more agents with $a^{I,J} = 1$ and every agent with $a^{I,J} = 0$ is in a neighborhood consisting of five or more agents with $a^{I,J} = 0$. However, when the equilibrium or steady state price is higher, say $(p_1/p_2) = 2$, then a steady state exists when every agent with $a^{I,J} = 1$ is in a neighborhood consisting of six or more agents with $a^{I,J} = 1$ and every agent with $a^{I,J} = 0$ is in a neighborhood consisting of four or more agents with $a^{I,J} = 0$. Because the rule for adapting preferences depends on average neighborhood consumption of good 1 the higher price of good 1 implies that more agents in a neighborhood must consume that good exclusively in order for total neighborhood consumption to exceed 50%.

Stating the conditions for a polarized steady state in terms of the total number of agents in a neighborhood with preferences equal to one highlights the connection with equilibria in spatial voting models where an agent's state next period is determined by a majority vote in his or her neighborhood this period. The equilibrium counting rules are shown with their corresponding price ranges:

$$\begin{aligned}
&(\text{R2}) \quad \frac{1}{8} < \frac{p_1}{p_2} \le \frac{2}{7} \quad \text{and} \quad \textstyle\sum_{\text{nbhd}_{(I,J)}} a_t^{i,j} \ge 2 \rightarrow a_t^{I,J} = 1 \quad \text{“two or more”}\\
&(\text{R3}) \quad \frac{2}{7} < \frac{p_1}{p_2} \le \frac{1}{2} \quad \text{and} \quad \textstyle\sum_{\text{nbhd}_{(I,J)}} a_t^{i,j} \ge 3 \rightarrow a_t^{I,J} = 1 \quad \text{“three or more”}\\
&(\text{R4}) \quad \frac{1}{2} < \frac{p_1}{p_2} \le \frac{4}{5} \quad \text{and} \quad \textstyle\sum_{\text{nbhd}_{(I,J)}} a_t^{i,j} \ge 4 \rightarrow a_t^{I,J} = 1 \quad \text{“four or more”}\\
&(\text{R5}) \quad \frac{4}{5} < \frac{p_1}{p_2} \le \frac{5}{4} \quad \text{and} \quad \textstyle\sum_{\text{nbhd}_{(I,J)}} a_t^{i,j} \ge 5 \rightarrow a_t^{I,J} = 1 \quad \text{“five or more”} \qquad (14)\\
&(\text{R6}) \quad \frac{5}{4} < \frac{p_1}{p_2} \le 2 \quad \text{and} \quad \textstyle\sum_{\text{nbhd}_{(I,J)}} a_t^{i,j} \ge 6 \rightarrow a_t^{I,J} = 1 \quad \text{“six or more”}\\
&(\text{R7}) \quad 2 < \frac{p_1}{p_2} \le \frac{7}{2} \quad \text{and} \quad \textstyle\sum_{\text{nbhd}_{(I,J)}} a_t^{i,j} \ge 7 \rightarrow a_t^{I,J} = 1 \quad \text{“seven or more”}\\
&(\text{R8}) \quad \frac{7}{2} < \frac{p_1}{p_2} \le 8 \quad \text{and} \quad \textstyle\sum_{\text{nbhd}_{(I,J)}} a_t^{i,j} \ge 8 \rightarrow a_t^{I,J} = 1 \quad \text{“eight or more”}
\end{aligned}$$

The comments “one or more”, “two or more”, etc. refer to the total number of agents in the neighborhood with preferences equal to one needed for $a^{I,J} = 1$, otherwise $a^{I,J} = 0$. These rules determine which spatial configurations of polarized preferences are consistent with an equilibrium. When the price falls strictly within the ranges indicated in (14) conditions C1 and C2 apply for all agents. However, when the steady state price falls on one of the endpoints of the price ranges indicated in (14), $p_1/p_2 \in \{1/8, 2/7, 1/2, 4/5, 5/4, 2, 7/2, 8\}$, then conditions C3 and C4 may apply for some agents.

5. Stability of polarized steady states

Examining the existence and characteristics of equilibria is the first step in the analysis of a dynamical system, however, examining the stability of equilibria establishes which steady states can actually arise as the result of the evolution of the system. A system will not evolve towards an unstable equilibrium; it is only possible to reach an equilibrium which is Lyapunov unstable by choosing that equilibrium as the initial condition. In contrast, equilibria which are Lyapunov stable are observable in simulation and in practice.

The crucial assumption in establishing the stability of a class of polarized equilibria involves $c_t^{I,J}$, the update term in the transition rule for preferences. When this term is strictly greater than zero in magnitude for all agents (e.g. when either condition C1 or C2 from (13) holds for all agents) then the steady state is stable in the sense of Lyapunov. Note that requiring each agent’s preferences to satisfy either C1 or C2 is the same as assuming a polarized steady state with an equilibrium price that does not fall on the endpoints of the price intervals in the rules listed in (14). Furthermore, the theorems established here apply to any lattice structure and neighborhood definition. [6]

Theorem 1. *Suppose that production exhibits constant returns to scale as in (1), (3) and (7). Let* $A_k = [a_k^{i,j}]$ *and let each element of* $A_* = [a_*^{i,j}]$ *satisfy either* C1 *or* C2 *as stated in (13). Then* A_* *is stable in the sense of Lyapunov.*

Proofs appear in the appendix. The key idea is that in a steady state the maximum possible change in $a_*^{I,J}$ is strictly positive for all $a_*^{I,J} = 1$ and strictly negative for all $a_*^{I,J} = 0$. If the size of the perturbation is bounded so that the sign of the update term does not change, then elements that were equal to one in equilibrium increase and elements that were equal to zero in equilibrium decrease and the entire state tends back towards the equilibrium.

Theorem 2. *Suppose that there is a fixed endowment of consumer goods as in (1) and (6). Let* $A_k = [a_k^{i,j}]$ *and let each element of* $A_* = [a_*^{i,j}]$ *satisfy either* C1 or C2. *Furthermore, suppose that* A_* *has at least one element equal to zero and at least one element equal to one. Then* A_* *is stable in the sense of Lyapunov.*

[6] Also, the theorems apply to steady states of model specifications with heterogeneous endowments. Eq. (11) must be defined appropriately but the steady state conditions stated in (13) remain the same. The existence of equilibria is not guaranteed, however, with heterogeneous endowments.

Theorem 3. *Suppose that production exhibits decreasing returns to scale as in (1), (4), (5) and (8). Let* $A_k = [a_k^{i,j}]$ *and let each element of* $A_* = [a_*^{i,j}]$ *satisfy either* C1 *or* C2. *Furthermore, suppose that* A_* *has at least one element equal to zero and at least one element equal to one. Then* A_* *is stable in the sense of Lyapunov.*

The instability of trivial equilibria composed exclusively of zeros or exclusively of ones is demonstrated in the simulation presented in Figs. 4 and 5 in the succeeding section. Starting from an initial condition where all agents have $a_0^{I,J} = 1$ save for one agent with $a_0^{I,J} = 0.999$, the lattice evolves to a steady state with almost equal numbers of consumers of the two goods.

5.1. Non-polarized steady states

Equilibria which do not consist exclusively of zero and one values of $a^{I,J}$ are easy to construct. For example, when $p_1/p_2 = 1$ each of the following are equilibria:

$$\begin{matrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 \end{matrix} \qquad \begin{matrix} 0.25 & 0.75 & 0.25 & 0.75 \\ 0.75 & 0.25 & 0.75 & 0.25 \\ 0.25 & 0.75 & 0.25 & 0.75 \\ 0.75 & 0.25 & 0.75 & 0.25 \end{matrix} \qquad \begin{matrix} 1 & 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 \\ 0 & 0.5 & 1 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5. \end{matrix} \tag{15}$$

The equilibrium condition C5 applies to at least one agent's preferences in any non-polarized steady state. The condition that the potential change in preferences must exactly equal zero (e.g. C3, C4, C5 from (14) applies to at least one agent) makes non-polarized steady states extremely sensitive to small perturbations. For instance, if any small amount ϵ is added to any agent's preferences in the steady states shown in (15), then the system diverges from (15) and ultimately converges to a polarized steady state. Non-polarized steady states are unstable and therefore will not be observed in simulations. Thus, starting from generic initial conditions, the system converges to a polarized steady state.

6. Evolution of average preferences and the price

This section considers the behavior of aggregates such as average preferences, the price, and the total output of the two goods. The interaction between the local interdependence of preferences and the specification of the production technology is explored through extensive computer simulations. Simulations are a standard research tool in the analysis of non-linear dynamical systems such as cellular automata and coupled map lattices, and are often used to investigate the relationship between initial conditions and global behaviors of the systems.

The set or region of the space of initial conditions that results in the same quantitative or qualitative outcome is called the basin of attraction for that outcome. Because agents' preferences are determined by a continuously valued parameter, there are an infinite number of initial conditions. (Recall that the initial choice of agents' preferences is the only random element in the system.) Due to the large size of the space of initial conditions, the simulation strategy utilized here relates an aggregate characteristic of the initial condition,

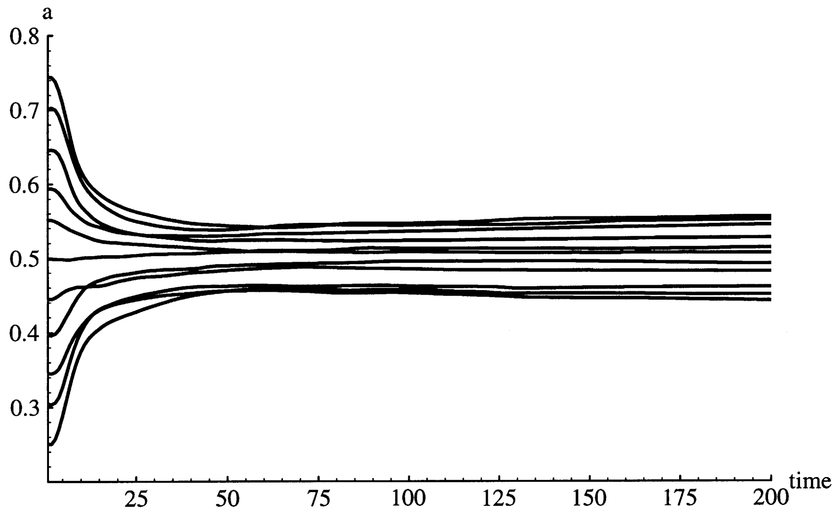

Fig. 3. Evolution of average preferences with exchange.

average initial preferences, to the same aggregate characteristic of the resulting steady state. Because the data is aggregated across agents, two initial conditions with the same average preferences may lead to different aggregate outcomes. However, as the succeeding discussion demonstrates, the evolution of aggregate preferences is surprisingly regular and in general depends much more crucially on the available production technology than on the initial state of preferences.

6.1. *Exchange Economy*

Fig. 3 presents the evolution of average preferences for 11 runs of the exchange economy with initial means ranging from 0.250 to 0.744. Agents have equal endowments of the two consumption goods, $e_1 = e_2 = 1$ and $N = 2500$.

Despite the differences in initial states, the evolution of preferences exhibits a strong tendency towards an equilibrium mean in a symmetric region about 0.5. The range of equilibrium average preferences observed in Fig. 3 correspond to the price interval for the “five or more” counting rule. A wide range of initial conditions with greatly varying average preferences fall in the basin of attraction for steady states with prices in the range $\{4/5, 5/4\}$.

This results directly from the functioning of the competitive market for goods: in each period the price operates as a negative feedback mechanism in the update rule and mediates the evolution of preferences in favor of the initially less popular good. Agents who prefer the relatively unpopular, and therefore low priced, good can consume more of it. The rule for updating preferences depends on the percent of total neighborhood consumption, as expressed in Eq. (9), consequently, these agents have a greater influence on local preferences than those who prefer the more popular, higher priced good. Next period, their neighbors will increase their preferences for the "unpopular" good. Over time the number of agents who prefer that good will increase, eventually pushing up the price and decreasing the influence of those agents on neighborhood preferences. Consequently, the evolution of average preferences tends towards a price of one, typically resulting in a steady state which conforms to the "five or more" counting rule that prevails in symmetric region around a price of one. In other words, the basin of attraction for steady states that conform to the "five or more" counting rule is large, and the basins of attraction for steady states which conform to other counting rules are relatively small.

This dynamic can be illustrated by considering an extreme example: an initial configuration of preferences that consists of $N - 1$ agents with $a_0^{I,J} = 1$ and one agent with $a_0^{I,J} = 0.999$. Fig. 4 shows the evolution of the lattice for this simulation.

In the first time period, the lone consumer of good 2 buys up the entire supply for virtually nothing. Consumption of good 2 in her neighborhood exceeds 50%, causing her neighbors to decrease their preferences for good 1. After purchasing large amounts of good 2, these agents transmit their newly acquired preferences for good 2 to their neighbors. At the end of the next period, the "fad" takes another step outwards, and so on in the succeeding periods. Fig. 5 shows the evolution of average preferences and the price. Consequently, with an exchange economy there are no initial conditions that converge to the trivial steady states composed entirely of zeroes or entirely of ones, other than the steady states themselves. In addition, these trivial steady are unstable in the sense of Lyapunov. [7]

The effects of varying the relative supply of the two goods in the exchange economy are shown in Fig. 6 which presents the evolution of average preferences for simulations of the exchange economy with relative endowments equal to 3:1; 2:1; 1:2; 1:3. The initial conditions for agents' preferences are the same as those used in simulation presented in Fig. 3. Note that average consumption necessarily reflects the relative supply of the two goods: if agents are endowed with twice as much good 1 as good 2 then overall twice as much good 1 will be consumed. Because agents have equal endowments, they have the same basic potential to influence neighborhood preferences. Combined with the price mechanism described above, this drives the average value of preferences towards the relative supply of good 1, $e_1/(e_1 + e_2)$.

The equilibria shown in Fig. 6 all correspond to the "four or more" and "six or more" counting rules (R4 and R6). The inertia of the bandwagon effect early in the evolution of preferences, when agents' preferences are heading towards zero and one, causes average preferences to become locked into equilibria of the "four or more" and "six or more" rules.

Clearly, allowing endowments to be heterogeneous across agents would affect the evolution of mean preferences, particularly if the size of agents' endowments and their preferences were somehow correlated in the initial state. While the form of the equilibrium conditions is the same, an equilibrium need not exist when endowments are allowed to vary.

[7] Note that the initial configuration can be considered a perturbation of the trivial steady state where $a_*^{I,J} = 1$ for all (I, J). The failure of the system to return to that steady state demonstrates its instability. Note that the dynamic would be the same no matter how small the perturbation away from $a_*^{I,J} = 1$.

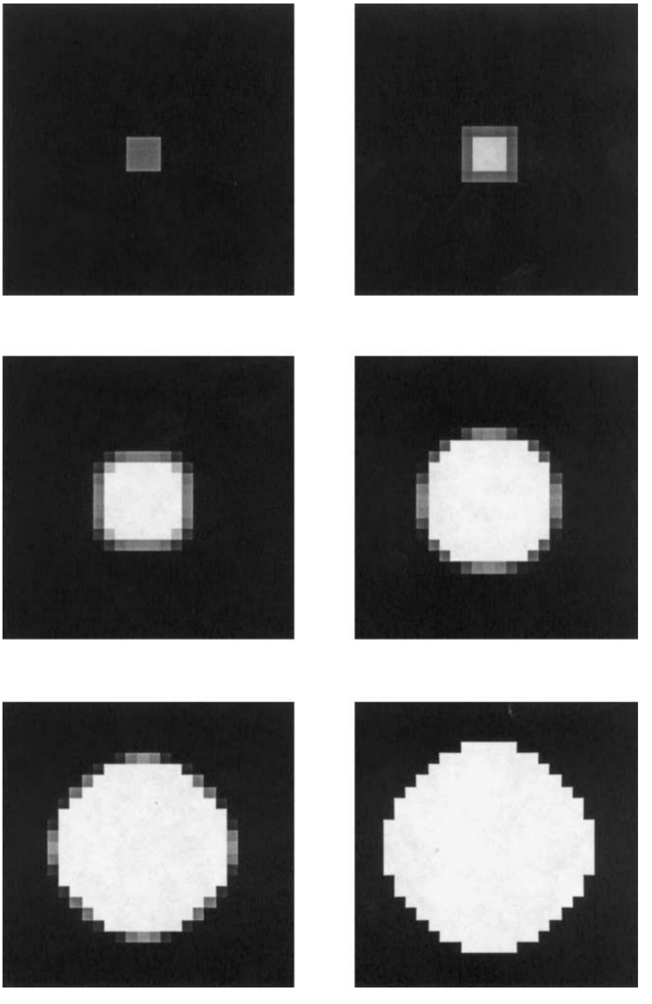

Fig. 4. Exchange economy with $a^{I,J} = 0.999$ for one agent.

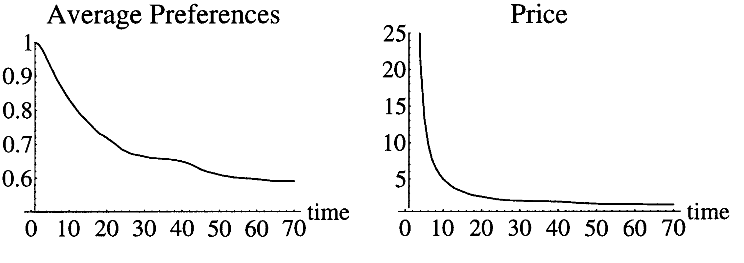

Fig. 5. Average preferences and price in 0.999 example.

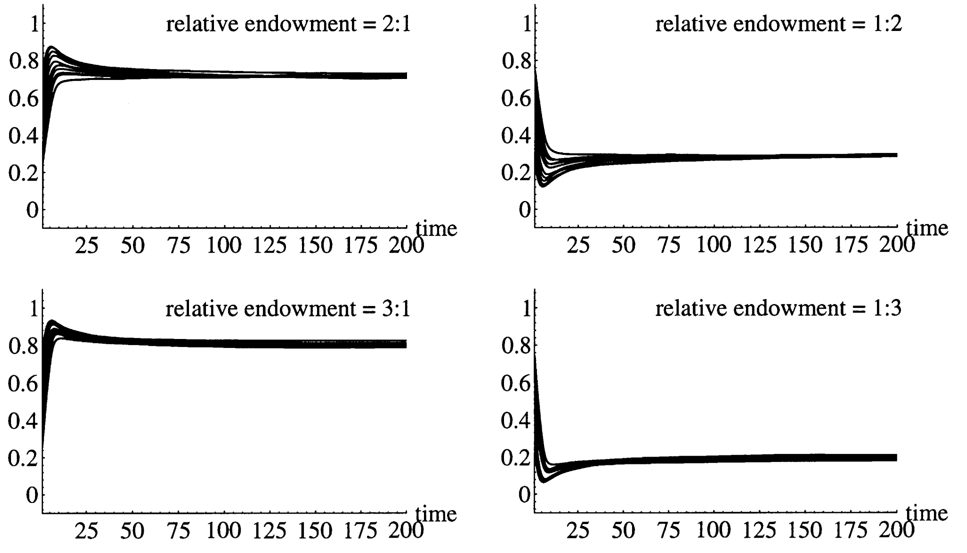

Fig. 6. Exchange economy with varying relative endowments.

6.2. Constant returns to scale

Fig. 7 presents the evolution of average preferences for 11 runs of the constant returns to scale economy with the same initial conditions for preferences used in Fig. 3 and with $p_1/p_2 = 1$.

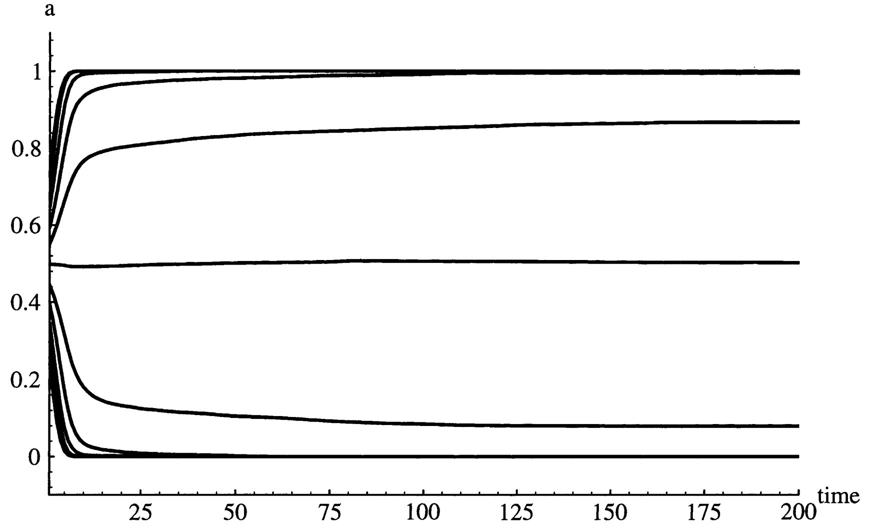

Fig. 7. Evolution of average preferences with CRS.

In contrast to the exchange economy, the constant returns to scale economy accentuates differences in the initial mean of preferences as the lattice evolves. Deviations of the initial state away from a mean of 0.5 are often compounded until the entire lattice consists entirely of ones or entirely of zeros. To understand this positive feedback mechanism note that in the constant returns to scale economy there are no global interactions in the update rule: the price does not depend on average preferences. The relative supply of the two goods varies as average preferences vary but the price remains constant. Because it is equally easy to produce both goods, $b_1 = b_2$, production places no constraints on the evolution of preferences. The update rule simply moves preferences in the direction of the neighborhood average, causing small differences in initial conditions to have a large effect on average preferences over time. Consequently, the basins of attraction for the trivial steady states composed of all zeroes or all ones are large with constant returns to scale production. The bandwagon effect in preferences completely determines the observed evolution of preferences.

However, when it is relatively easier to produce one of the goods, or, equivalently, when the price is not equal to one, production does influence the equilibrium average of preferences. Fig. 8 presents the evolution of average preferences when $p_1/p_2 = 1.1$, 1.2 and 1.3 for five different initial conditions with an initial mean of preferences near 0.5. As was observed in the exchange economy, the higher price of good 1 weights the evolution of preferences in favor of good 2.

6.3. *Decreasing returns to scale*

The decreasing returns to scale economy represents an intermediate case between constant returns to scale and exchange. Fig. 9 presents simulation results for this model. With decreasing returns to scale production, the supply of goods responds to changes in agents' preferences, but the cost of producing additional goods depends positively on the level of production. The price still affects the evolution of preferences as described for the exchange economy but not as strongly because some of the change in aggregate demand is offset by increases in supply. Because firms have access to identical DRS technology, the maximum amount of goods will be produced when production is shared equally between the two firms. This occurs when average preferences equal 0.5 and the price is one. The price dynamics described above for the exchange economy operate here as well, driving average preferences towards 0.5. Consequently, the greater the cost in terms of output of dividing production unequally between the two firms or industries, the greater the tendency for average preferences to converge to 0.5 and the larger the basin of attraction of those steady states.

In an exchange economy, the evolution of preferences favors steady states in which average preferences reflect the relative availability of goods, despite wide variations in initial states. In contrast, with constant returns to scale production and a price of one, small deviations in initial average preferences can play a decisive role in the final outcome. When one of the goods can be produced at a lower cost (with fewer inputs) the evolution of average preferences favors the less costly good and is more likely to converge to a trivial steady state where only the cheaper good is consumed. When production exhibits decreasing returns to scale, the evolution of aggregate preferences tends to maximize the total quantity of goods produced, or alternatively, tends towards a price of one.

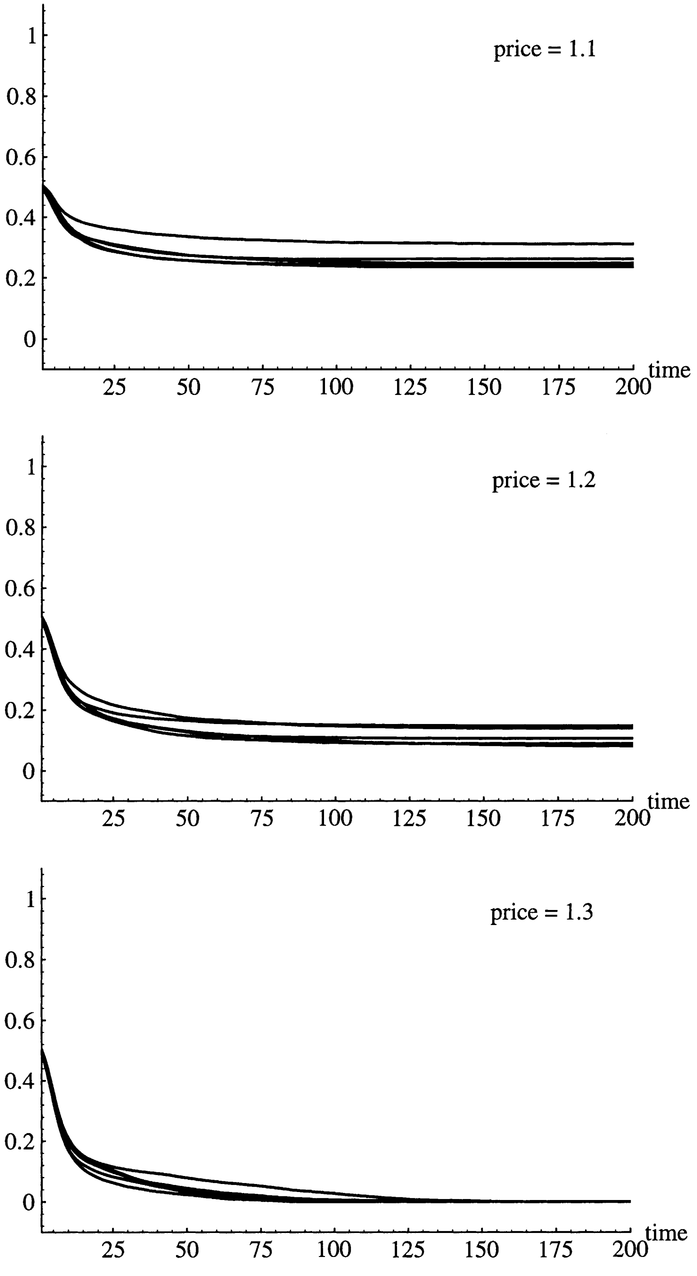

Fig. 8. Evolution of average preferences with CRS, $p_1/p_2 = 1.1, 1.2, 1.3$.

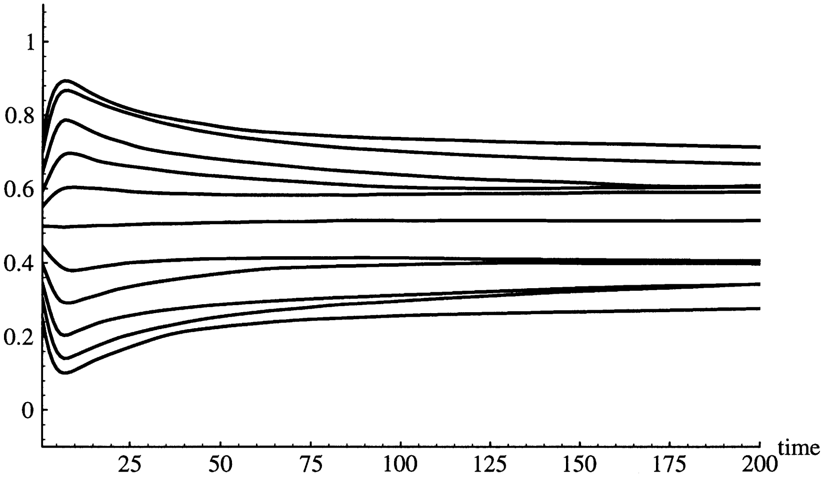

Fig. 9. Evolution of average preferences with DRS.

The key to understanding the aggregate behavior of preferences lies in the general equilibrium market structure; the global interactions reflected in the market clearing price determine the global evolution of the system. Agents who prefer the cheaper good have a larger influence on neighborhood consumption, consequently, the elasticity of supply with respect to price has a greater influence than the initial state of preferences. Models of imitative behavior that abstract away from a market clearing framework thus risk ignoring a crucial determinant of aggregate behavior: prices.

7. Strategic behavior

Because there is no storage of goods and no credit markets, the only way agents' behavior today can affect their future utility is through their influence on total neighborhood consumption and hence on the future trajectory of preferences. In order to account for this influence agents would need to predict not only all future prices and all future states of preferences, but also the strategies (consumption plans) of all agents. Such a computation is often not feasible even when all the relevant information is readily available. [8]

However, the optimal strategy of "fully rational" agents who consider the effects of their own consumption on the evolution of preferences can be examined in a much simpler framework. Suppose that in addition to maximizing utility period by period, agents are able to choose the initial conditions of their preferences. In other words, instead of let-

[8] Note that many cellular automata, which are related dynamical systems that only involve local interactions, have been shown to be computationally irreducible: the most efficient method of determining the behavior of such systems is explicitly calculating the trajectory. Any "reduced form" solution actually involves more computation. Consequently, the explicit calculation of trajectories and basins of attraction constitute the most viable solution technique for these dynamical systems.

ting agents indirectly influence their future preferences through their current consumption choices, let agents choose their preferences outright in the first period with full knowledge of the structure of the economy and the rules that govern the evolution. Examining agents' optimal behavior in this context demonstrates the steady states consistent with full rationality without necessitating a full account of the transient behavior of the system.

Furthermore, for simplicity suppose that there is constant returns to scale production, i.e. that the price is fixed. For any given price and endowment of labor, the utility function $x_1^{a_0} x_2^{1-a_0}$ achieves a maximum at either $a_0 = 1, x_2 = 0$ or $a_0 = 0$, $x_1 = 0$. Clearly, if the price is greater than 1, the outcome where all agents choose $a_0 = 0$ and consume x_2 exclusively is a Nash equilibrium because it provides the highest feasible utility for every agent. Similarly, if the second good is more expensive then all agents consuming good 1 exclusively is also a Nash equilibrium. If the price is exactly equal to one, then any stable configuration of preferences is a Nash equilibrium. To see this, note that whenever any agent has preferences not equal to zero or one they receive lower utility. Hence, an initial choice of preferences in a stable configuration maximizes all agents' lifetime utility. Similar arguments apply with an exchange economy and decreasing returns to scale, keeping in mind that all agents' choice of preferences affect the price. [9]

The major conclusion of the previous section was that average preferences evolve towards the good which is relatively abundant or which is relatively cheaper to produce. Average preferences tend towards steady states with a price near one. To be more precise, the basins of attractions for equilibria that are close to the Nash equilibria that maximize the total utility of all agents are large and hence observed frequently. While the system can evolve to steady states with a price far from one that have lower total utility, the basins of attraction for such equilibria are relatively small and the equilibria are correspondingly rare. Even though agents are engaging in strictly imitative behavior and not accounting for the effect of their consumption on the future state of preferences, the negative feedback of the price mechanism favors goods which are relatively cheaper and moves the system in the direction of a stable configuration of preferences consistent with the Nash equilibria described above.

8. Conclusion

This paper explores the behavior of a system of interdependent preferences in the context of a competitive general equilibrium environment. Both the positive feedback of the bandwagon effect and the negative feedback of the price mechanism play an important role in shaping individual and aggregate behavior over time. The most important microeconomic characteristic of these models is the polarization of preferences and consumption, and the tendency of like-minded agents to form stable clusters in equilibrium. The Lyapunov

[9] Note that these need not be the only Nash equilibria. For example, if all agents on the lattice have chosen initial preferences which favor the more expensive good and no choice of individual consumption can move neighborhood preferences towards the cheaper good, then the optimal response would be to choose preferences that favor the more expensive good.

stability of certain kinds of polarized equilibria is also established. The most important macroeconomic characteristic of these models is the tendency of average preferences to evolve so as to reflect the relative availability of goods or to minimize the per unit cost of production for both goods (equivalently, to maximize the total output produced from a fixed endowment of input goods). Empirical applications of these models may include analysis of the diffusion of technologies with network externalities such as computer operating systems and of behaviors which exhibit strong spatial correlations such as the regional variations in medical treatments.

Acknowledgements

I thank Steven Durlauf, William Brock, Blake LeBaron, Larry Samuelson and a multitude of seminar participants and anonymous referees for helpful comments.

Appendix A. Proofs of theorems

A.1. Proof of Theorem 1

First note that: $|c_*^{I,J}| = \left| r\left(\sum_{\text{nbhd}_{(I,J)}} a_t^{i,j} / \left(\sum_{\text{nbhd}_{(I,J)}} a_t^{i,j} + (p_1/p_2)\sum_{\text{nbhd}_{(I,J)}} (1-a_t^{i,j})\right) - 0.5\right)\right| > 0$ for all (i, j). This follows from the requirement that p_1/p_2 fall in an *open* interval for each counting rule. For any given value of $\sum_{\text{nbhd}_{(I,J)}} a_*^{i,j}$, $|c_*^{I,J}|$ is a continuous function with a unique minimum of 0 at $(p_1/p_2) = \sum_{\text{nbhd}_{(I,J)}} a_t^{i,j} / \sum_{\text{nbhd}_{(I,J)}} (1-a_t^{i,j})$. For the relevant integer values of $\sum_{\text{nbhd}_{(I,J)}} a_*^{i,j}$ the minima occur at the endpoints of the counting rule intervals, {1/8, 2/7, 1/2, 4/5, 5/4, 2, 7/2, 8}. To simplify the notation, let $\bar{p} = 9(p_1/p_2)$.

Consider the function $V(A_k) = \sum_{i,j} |a_*^{i,j} - a_k^{i,j}|$. We shall show that $V(A)$ is a Lyapunov function, thus establishing the stability of the system by Lyapunov's Theorem (Luenberger, 1979). In order to be a Lyapunov function, $V(A)$ must fulfill three conditions.

1. $V(A)$ has a unique minimum at $V(A_*)$.
   $V(A_*) = \sum_{i,j} |a_*^{i,j} - a_*^{i,j}| = 0$ and $V(A) \geq 0\, \forall a_K^{i,j} \neq a_*^{i,j}$.
2. $V(A)$ is continuous.
   $V(A)$ is a sum of continuous functions, and hence is continuous.
3. $V(A_{k+1}) \leq V(A_k)$ for all A_k such that $||A_k - A_*|| < \Delta$, for some $\Delta > 0$.

Let $||A_k - A_*|| = \sum_{i,j} |a_k^{i,j} - a_*^{i,j}|$, and let $\delta_k^{i,j} = a_k^{i,j} - a_*^{i,j}$. For notational convenience, define $\bar{\delta}_{I,J} = \sum_{\text{nbhd}_{(I,J)}} \delta^{i,j}$ and $\bar{a}_{I,J} = \sum_{\text{nbhd}_{(I,J)}} a^{i,j}$. Choose Δ such that

$$\min_{i,j} |c_*^{I,J}| > |c_*^{I,J} - c_k^{I,J}| \tag{A.1}$$

or

$$\min_{i,j}|c_*^{I,J}| > r\left|\frac{\bar{p}\bar{\delta}_{I,J}}{\left(\bar{a}_{I,J}+\bar{p}-(1/9)\bar{p}\bar{a}_{I,J}\right)\left(-\bar{a}_{I,J}-\bar{\delta}_{I,J}-\bar{p}+(1/9)\bar{p}\bar{a}_{I,J}+(1/9)\bar{p}\bar{\delta}_{I,J}\right)}\right| \tag{A.2}$$

for all neighborhoods (I, J) and for all possible perturbations $\delta^{i,j}$ such that $\left|\sum_{\mathrm{nbhd}_{(I,J)}}\delta^{i,j}\right| < \Delta$. Such a Δ exists because

$$\lim_{|\bar{\delta}_{I,J}|\to 0} r\left|\frac{\bar{p}\bar{\delta}_{I,J}}{\left(\bar{a}_{I,J}+\bar{p}-(1/9)\bar{p}\bar{a}_{I,J}\right)\left(-\bar{a}_{I,J}-\bar{\delta}_{I,J}-\bar{p}+(1/9)\bar{p}\bar{a}_{I,J}+(1/9)\bar{p}\bar{\delta}_{I,J}\right)}\right| = 0 \tag{A.3}$$

which implies that for any $\Delta > 0$ there exists a $\left|\sum_{\mathrm{nbhd}_{(I,J)}}\delta^{i,j}\right| > 0$ such that $|c_*^{i,j}-c_k^{i,j}|<\Delta$ whenever $\left|\sum_{\mathrm{nbhd}_{(I,J)}}\delta^{i,j}\right| < \Delta$.

Note that Eq. (A.1) implies that $c_k^{I,J}$ and $c_*^{I,J}$ have the same sign. If $a_*^{I,J} = 0$ then $c_*^{I,J} < 0$ and $c_k^{I,J} < 0$. Thus, $0 \le a_{k+1}^{I,J} \le a_k^{I,J}$. If $a_*^{I,J} = 1$ then $c_*^{I,J} > 0$ and $c_k^{I,J} > 0$. Thus, $1 \ge a_{k+1}^{I,J} \ge a_k^{I,J}$. Together these imply $|a_k^{I,J} - a_*^{I,J}| \ge |a_{k+1}^{I,J} - a_*^{I,J}|$ for all (i, j) with the inequality holding strictly for at least one element whenever $A_k \neq A_*$. Therefore, $V(A_{k+1}) < V(A_k)$ for all $A_k \neq A_*$.

A.2. Proof of Theorem 2

Proof. Let $\bar{A}_* = \sum_{i,j} a_*^{i,j}$, $\bar{D} = \sum_{i,j}\delta_k^{i,j}$ and $N = n^2$. The proof of Theorem 2 follows the proof of Theorem 1 above with Eq. (A.2) replaced with:

$$\min_{i,j}|c_*^{I,J}| > r\left|\frac{\bar{a}_{I,J}}{\bar{a}_{I,J}+(e_1\bar{A}_*(9-\bar{a}_{I,J}))/(e_2(N-\bar{A}_*))} - \frac{\bar{a}_{I,J}+\bar{\delta}_{I,J}}{\bar{a}_{I,J}+\bar{\delta}_{I,J}+((9-\bar{a}_{I,J}-\bar{\delta}_{I,J})(\bar{D}+e_1\bar{A}_*)/(e_2(N-\bar{A}_*-\bar{D})))}\right| \tag{A.4}$$

and Eq. (A.3) replaced with:

$$\lim_{|\bar{D}|\to 0} r\left|\frac{\bar{a}_{I,J}}{\bar{a}_{I,J}+(e_1\bar{A}_*(9-\bar{a}_{I,J}))/(e_2(N-\bar{A}_*))} - \frac{\bar{a}_{I,J}+\bar{\delta}_{I,J}}{\bar{a}_{I,J}+\bar{\delta}_{I,J}+((9-\bar{a}_{I,J}-\bar{\delta}_{I,J})(\bar{D}+e_1\bar{A}_*)/(e_2(N-\bar{A}_*-\bar{D})))}\right| = 0. \tag{A.5}$$

The limit of the previous expression can be verified by combining the terms on the left-hand side of Eq. (A.5) over a common denominator and noting that the limit of the numerator is zero while the limit of the denominator is a positive constant. □

A.3. Proof of Theorem 3

Proof. The proof of Theorem 3 follows the proof of Theorem 1 above with Eq. (A.2) replaced with:

$$\min_{i,j}|c_*^{I,J}| > r \left| \frac{\bar{a}_{I,J}}{\bar{a}_{I,J} + (9 - \bar{a}_{I,J})\sqrt{\bar{A}_*/(N - \bar{A}_*)}} - \frac{\bar{a}_{I,J} + \bar{\delta}_{I,J}}{\bar{a}_{I,J} + \bar{\delta}_{I,J} + (9 - \bar{a}_{I,J} - \bar{\delta}_{I,J})\sqrt{(\bar{A}_* + \bar{D})/(N - \bar{A}_* - \bar{D})}} \right| \tag{A.6}$$

and Eq. (A.3) replaced with:

$$\lim_{|\bar{D}| \to 0} r \left| \frac{\bar{a}_{I,J}}{\bar{a}_{I,J} + (9 - \bar{a}_{I,J})\sqrt{\bar{A}_*/(N - \bar{A}_*)}} - \frac{\bar{a}_{I,J} + \bar{\delta}_{I,J}}{\bar{a}_{I,J} + \bar{\delta}_{I,J} + (9 - \bar{a}_{I,J} - \bar{\delta}_{I,J})\sqrt{(\bar{A}_* + \bar{D})/(N - \bar{A}_* - \bar{D})}} \right| = 0 \tag{A.7}$$

The limit of the previous expression can be verified by combining the terms on the left-hand side of Eq. (A.7) over a common denominator and noting that the limit of the numerator is zero while the limit of the denominator is a positive constant. □

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